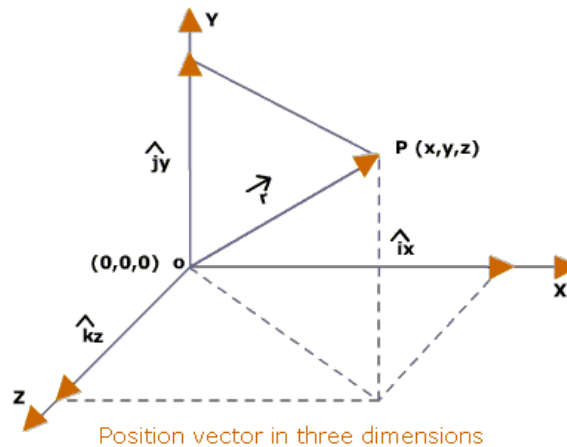


# Kinematics

# Position

To specify a position vector you need to specify:

- Origin
- Distance
- Direction



$$\mathbf{r}_A = (x_A, y_A, z_A),$$

$$|\mathbf{r}| = \sqrt{x_A^2 + y_A^2 + z_A^2}.$$

If using a 3D right-handed coordinate system with the origin being the reference point for the position vector, it is enough to specify the coordinates  $x$ ,  $y$  and  $z$ .

For a moving object the position vector is a function of time.

# Velocity & Acceleration

- Velocity is defined as the change in position over a change in time; thus the average velocity is  $\bar{v} = \frac{\Delta r}{\Delta t}$ , and the instantaneous velocity is  $v = \lim_{\Delta t \rightarrow 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$ ,
- For motion in one dimension the velocity is the slope of the position line plotted versus time.
- The same logic is used in deriving the average and instantaneous acceleration resulting in:

$$\bar{a} = \frac{\Delta v}{\Delta t}, \quad a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt},$$

- Jerk is defined as the rate of change of acceleration:  $\vec{j} = \frac{d\vec{a}}{dt} = \frac{d^2\vec{v}}{dt^2} = \frac{d^3\vec{s}}{dt^3}$

# Example 1

- A body starts from rest at  $x=0$ .  $a_x(t)=2t-4$  [m/s<sup>2</sup>]. Find the jerk and the position as a function of time.

Solution:

At rest  $\Rightarrow v_{x0}=0$ ; At  $x=0 \Rightarrow x_0=0$

Integrating:

$$v_x(t)=v_{x0}+t^2-4t=t^2-4t \text{ [m/s]}$$

$$x(t)=x_0+(t^3/3)-2t^2=(t^3/3)-2t^2 \text{ [m];}$$

$$j_x=da_x/dt=2 \text{ [m/s}^3\text{]}$$

# Example 2

A body is moving along  $x$  with a constant jerk. At  $t=2s$ , its velocity is  $4$  m/s. At  $t=4.5s$  and  $t=5s$ , its acceleration is respectively  $2.1$  m/s<sup>2</sup> and  $4$  m/s<sup>2</sup>. At  $t=1s$  it is at  $x=3.4$ m. Determine the position of the body at  $t=7s$ .

$$x = x_0 + v_0 t + \frac{a_0 t^2}{2} + \frac{j t^3}{6}$$

@  $4.5s$   $a(t) = a_0 + jt = a_0 + 4.5j = 2.1$   
@  $5s$   $a(t) = a_0 + 5j = 4$

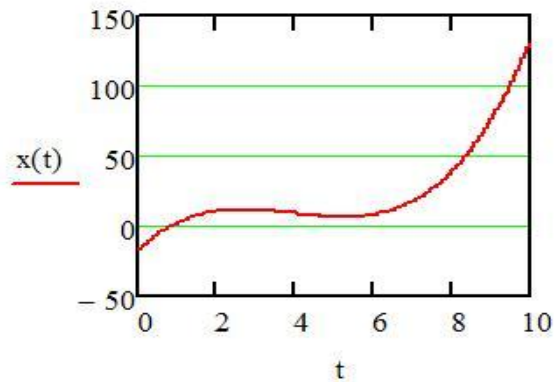
$$\begin{array}{r} a_0 + 4.5j = 2.1 \\ - \\ a_0 + 5j = 4 \\ \hline -0.5j = -1.9 \\ j = 3.8 \text{ m/s}^3 \\ \Rightarrow a_0 = 2.1 - 4.5j = -15 \text{ m/s}^2 \end{array}$$
$$v(t) = v_0 + a_0 t + \frac{j t^2}{2}$$
$$v(2) = v_0 + (-15)(2) + \frac{3.8 \times 2^2}{2} = 4$$
$$v_0 = 26.4 \text{ m/s}$$
$$x(t) = x_0 + v_0 t + \frac{a_0 t^2}{2} + \frac{j t^3}{6}$$
$$x(1) = x_0 + 26.4 \times 1 + \frac{(-15)(1)^2}{2} + \frac{3.8 \times 1^3}{6} = 3.4$$
$$x_0 = -16.13 \text{ m}$$
$$\Rightarrow x(7) = -16.13 + 26.4 \times 7 + \frac{(-15)(7)^2}{2} + \frac{3.8 \times 7^3}{6} = 17.763$$

# Example 2

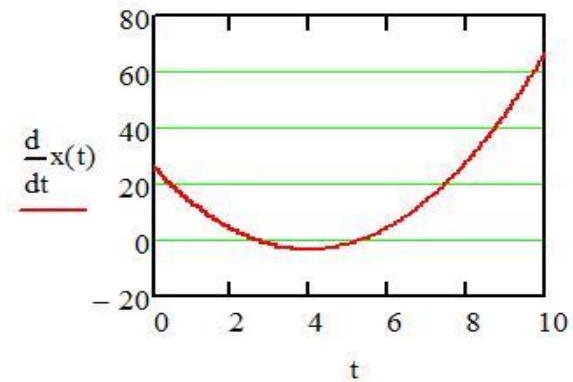
$$x(t) := -16.77 + 26.4 \cdot t - \frac{15 \cdot t^2}{2} + \frac{3.8 \cdot t^3}{6}$$

$$x(7) = 17.763$$

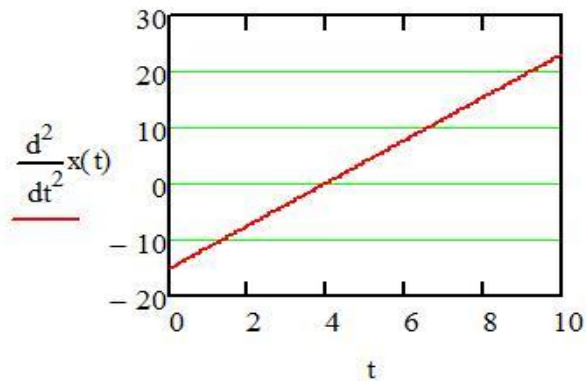
Position vs time:



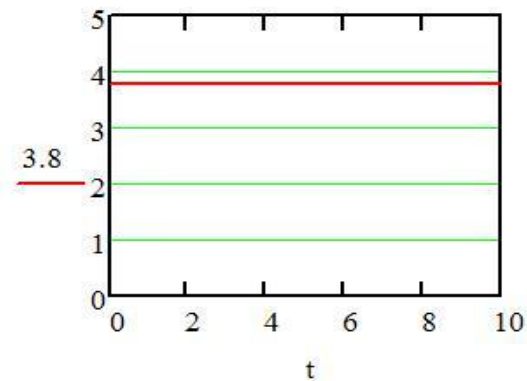
Velocity vs time:



Acceleration vs time:

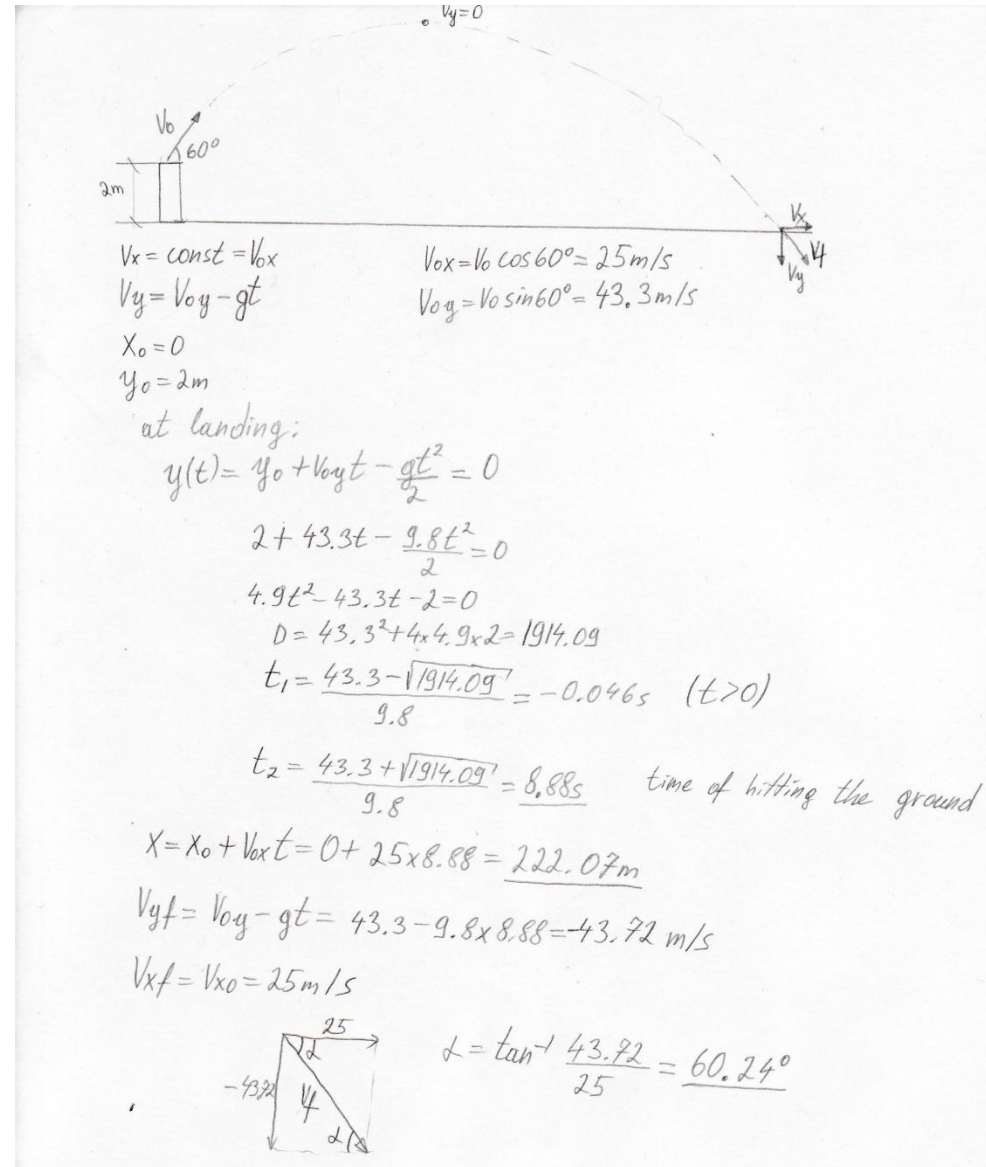


Jerk vs time:



# Example 3

A body is launched with an initial speed of 50m/s at an angle of 60 degrees with the horizontal from a height of 2m. How far from its initial position will it land? What will be the angle it makes with the horizontal when hitting the ground?



# Relative Velocity

- The position of one body with respect to another is described by its relative position vector, equal to the difference between the position vectors, relative to the origin, of the two bodies:  $\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B = \mathbf{r}_A - \mathbf{r}_O - (\mathbf{r}_B - \mathbf{r}_O) = \mathbf{r}_{A/O} - \mathbf{r}_{B/O}$  .

This equation can be rewritten as:  $\mathbf{r}_{A/O} = \mathbf{r}_{A/B} + \mathbf{r}_{B/O}$  ,

Differentiating with respect to  $t$ , we obtain:

$$\mathbf{V}_A = \mathbf{V}_B + \mathbf{V}_{A/B}. \quad \mathbf{V}_{A/B} = \mathbf{V}_A - \mathbf{V}_B.$$



# Example

- An airplane is moving at a constant speed, equal to  $300\mathbf{i}+200\mathbf{j}+75\mathbf{k}$  with respect to a control tower. A jetfighter is moving at a constant speed, equal to  $750\mathbf{i}-320\mathbf{j}+0\mathbf{k}$  with respect to the same control tower. Determine the speed of the airplane with respect to the jetfighter.

$$\begin{aligned}V_{A/J} &= V_{A/C} - V_{J/C} = (300-750)\mathbf{i} + [200-(-320)]\mathbf{j} + (75-0)\mathbf{k} = \\ &= -450\mathbf{i} + 520\mathbf{j} + 75\mathbf{k}\end{aligned}$$

# Rotational Motion

The relationships stated for linear motion are also valid for rotational motion, where

- The angular position,  $\theta$ , corresponds to the position vector  $r$
- The angular speed,  $\omega$ , corresponds to the velocity:  $\omega = \frac{d\theta}{dt}$
- The angular acceleration,  $\alpha$ , is analogous to the linear acceleration:  $\alpha = \frac{d\omega}{dt}$

# Cable Constraints

## Assumptions:

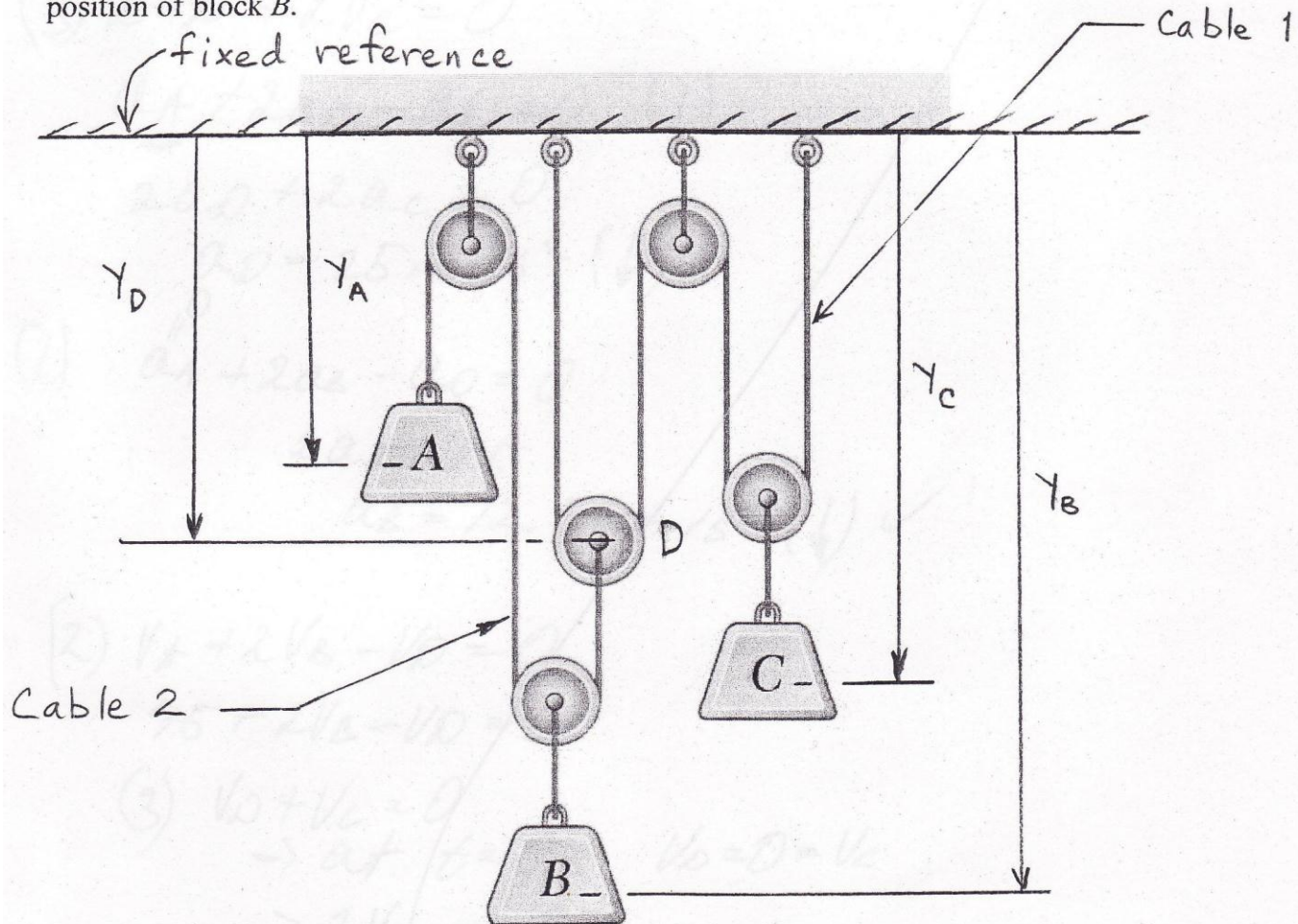
- Idealized cord
- Remains in tension
- Undeformable

## Methodology:

- Select a reference point/line to use in expressing position vectors
- Using the position vectors of different points on the cable express its length
- The length of the cable must remain constant
- Differentiate the equation to obtain expressions for the velocity and acceleration. Note that the derivative of the constant length of the cable yields zero

# Example

Block  $C$  starts from rest at  $t = 0$  and moves upward with a constant acceleration of  $25 \text{ mm/s}^2$ . Knowing that block  $A$  moves downward with a constant velocity of  $75 \text{ mm/s}$ , determine (a) the time for which the velocity of block  $B$  is zero, (b) the corresponding position of block  $B$ .



# Example Solution

Considering the given reference frame and position vectors  
 at  $t=0s$   $a_c = -25 \text{ mm/s}^2$  ( $\uparrow$ )  $V_A = 75 \text{ mm/s}$  ( $\downarrow$ )  $a_A = 0$   
 $V_c = 0$  /from rest/ /constant velocity/

Constraints:

$$y_A + y_B + y_{B/D} = \text{constant}$$

$$2y_D + 2y_C = \text{constant}$$

differentiate:

$$V_A + V_B + V_{B/D} = 0 ; V_{B/D} = V_B - V_D \Rightarrow V_A + 2V_B - V_D = 0$$

$$2V_D + 2V_C = 0$$

$$a_A + 2a_B - a_D = 0$$

$$2a_D + 2a_C = 0 ; a_C = -25 \text{ mm/s}^2$$

$$\rightarrow 2a_D - 2 \times 25 = 0$$

$$a_D = 25 \text{ mm/s}^2 (\downarrow)$$

$$a_A + 2a_B - a_D = 0 ; a_A = 0 , a_D = 25 \text{ mm/s}^2$$

$$2a_B - 25 = 0$$

$$a_B = 12.5 \text{ mm/s}^2 (\downarrow)$$

$$2V_D + 2V_C = 0 ; V_C = 0 \Rightarrow V_D = 0$$

$$V_A + 2V_B - V_D = 0 ; V_A = 75 \text{ mm/s} , V_D = 0$$

$$75 + 2V_B - 0 = 0$$

$$a) \quad V_B = -37.5 \text{ mm/s} (\uparrow) \text{ at } t=0s$$

$$V_{B, \text{final}} = V_B + a_B \Delta t = 0$$

$$-37.5 + 12.5 \Delta t = 0$$

$$\Delta t = 3s$$

$$t = t_0 + \Delta t = 0 + 3 = 3s$$

$$b) \quad y_B = y_{B_0} + V_{B_0} \Delta t + \frac{a_B \Delta t^2}{2} = y_{B_0} + (-37.5)(3) + \frac{(12.5)(3)^2}{2} =$$

$$= y_{B_0} - 56.25 \text{ mm} ,$$

where  $y_{B_0}$  is the initial position of B at  $t=0$ .